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THEORETICAL RESEARCH REPORT 1/46

The Theory of Wedge Penetration at Oblique Incidence and its application to the calculation of forces on a yawed shot impacting on armour plate at any angle

by

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Summary

The forces on a shot entering armour plate, at any angle of incidence and any angle of yaw, are calculated by an approximate method. The approximation is based on the solution of the associated plastic problem of oblique penetration by a wedge. Full account is taken of the formation of a coronet or lip, and of the resistance which this offers to the shot.

A full numerical solution is given for a wedge of 30° semi-angle. As the obliquity increases it is found that the axial component of resistance increases for the same projectile travel. The lateral component, zero for normal penetration, increases more rapidly and overtakes the axial component at just under 30° obliquity.

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1. Introduction

The object of this report is to attempt to calculate the forces acting over the nose of a projectile as it enters its target. An accurate knowledge of these forces would be valuable for many purposes: for example, the determination of decelerations for fuse design; the calculation of the turn of a shot on impact; and the more complete understanding of the shatter phenomenon.

We shall evaluate an approximation to the forces on a shot entering a plate at any angle of incidence and with any amount of yaw, by solving the associated problem of the penetration of a wedge. It is assumed that the deformation is ductile while the nose is entering and that the plate is sufficiently thick for no back bulge to be formed during this stage of the penetration. It is necessary, in a preliminary investigation of a subject of such complexity, to limit the problem to finding the static forces on the shot, corresponding to static punching at any angle of attack and orientation. That this is not a serious limitation on the utility of the results over a considerable velocity range is shown by the close correlation found by Dr. Baines between static punching tests and the actual firing results in partial penetration at normal incidence.

It is of course true that the proportional contribution to the resistance by the forces due to the inertia of the plate material is greatest in the initial stages of penetration, and that this increases with striking velocity, resulting in set-up and finally shatter. But even in those cases when the inertial forces are too great to be ignored in determining the absolute magnitude of the forces on the shot, yet a comparison of static forces for different angles of attack is still useful. For it is well known that the effect of increasing the obliquity is markedly to decrease the shatter velocity. The inference would seem to be that, in angle attack, the static forces by themselves are tending to bring the shot nearer to the point of rupture, so that a smaller inertial force or striking velocity suffices to bring the stress in the nose to the critical rupture value. If the shot were not sufficiently hardened, the static forces alone would presumably be enough to break the head.

It will also be of great interest to correlate the calculations with experimental values found from static punching at oblique incidence. It is understood that Dr. Baines is developing the technique for such tests.

2. Wedge penetration with completely plastic coronet

At the present stage of development of the theory of plasticity it is still necessary, in order to get a solution, to reduce a complex 3-dimensional problem to one of plane stress or strain. For this reason we consider penetration of a semi-infinite medium by a long wedge. This is a plane 2-dimensional problem in which there is no displacement in a direction perpendicular to the plane i.e. in the direction along the length of the wedge. Though this apparently represents a rather severe idealisation of actual conditions, it should be noted that the magnitudes of the resisting pressures on a 2-dimensional wedge and on a 3-dimensional cone are roughly the same at normal incidence (Ref. 1). This holds good with the exception of long thin heads or very blunt ones, neither of which are cases of practical importance in attack of thick armour. Moreover the mechanism of penetration is the same in both cases: that of pushing the plate material aside and upwards.

For added simplicity the wedge is taken to have straight sides, which provides a satisfactory approximation for any pointed head shape during the initial stages of penetration, so that, provided the plate is sufficiently thick, the configuration is always similar at any depth of penetration. In Fig.1 the wedge is constrained by applied external forces to move along the straight line OP, the direction of penetration. θ is the angle between OP and the normal ON to the target surface. For comparison with actual firings θ can be taken to be the angle of attack since we are calculating the forces in the first stages of entry, before the plate resistance has overcome the inertia of the shot to produce any substantial turning. The axis of symmetry OA of the wedge (in the plane of the paper) is constrained to lie at a fixed angle of yaw ϵ with the direction of motion OP. ϵ if positive of OA has rotated through an anticlockwise angle from OP i.e., away from the side of the wedge nearest to the target surface.*

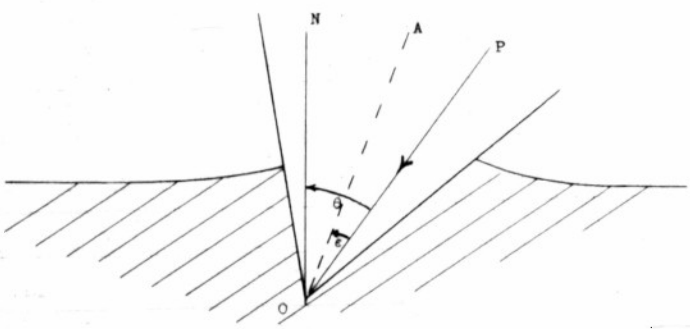

Fig.1

The theory of wedge penetration at normal incidence without yaw was worked out in Ref.1 by the present authors. As we are primarily concerned in this report with practical applications, we shall derive the required equations very briefly. Apart from one new feature, which will be described fully in the Appendix, the method of derivation is exactly the same, and if further details are required the first report should be consulted.

When the angle of incidence θ is not too great (this will be made more precise in Section 3), the lip or coronet of the displaced material

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* It is assumed that $|\epsilon| < \beta$ so that the wedge makes contact with the target on both sides. It is perfectly easy to consider the case of $|\epsilon| > \beta$ but this is of little practical significance.

is completely plastic and takes the form shown shaded in Fig.2. Material not shaded has only been subjected to elastic straining and the displacements are negligible.

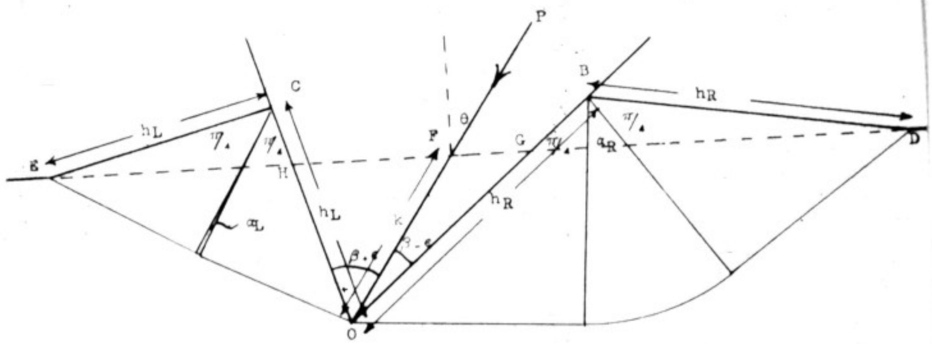

Fig.2

Both displaced surfaces are straight and BD = OB = h_R, EC = OC = h_L. If the wedge angle is 2β and the yaw is ϵ, then angle FOB = $\beta - \epsilon$, angle FOC = $\beta + \epsilon$. The configuration remains similar as the penetration OF = k increases. Material initially lying to the right of OP is displaced into the right-hand coronet, and since the material is considered incompressible* area BGD = area OFG; similarly area CHE = area OHI. The angles OBD, OCE are in general different; using the same notation as in Ref.1. we write OBD = $\pi/2 + \alpha_R$, OCE = $\pi/2 + \alpha_L$.

Then for the right-hand lip (4.1) of Ref.1 becomes

$$h_R \cos(\beta + \theta - \epsilon) - k \cos\theta = h_R \sin(\beta + \theta - \epsilon - \alpha_R) \quad \ldots\ldots. (2.1)$$

(4.3) of Ref.1 becomes

$$h_R \cos\alpha_R = k\left[\sin(\beta - \epsilon) + \cos(\beta - \epsilon - \alpha_R)\right] \quad \ldots\ldots. (2.2)$$

Eliminating h_R/k :-

$$\cos(2\beta + \theta - 2\epsilon - \alpha_R) = \sin\theta + \frac{\cos\theta\cos\alpha_R}{1 + \sin\alpha_R} \quad \ldots\ldots. (2.3)$$

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* Elastic strains are neglected in comparison with plastic strains since the material is free to flow out at the surface.

Hence given β, θ and ϵ we can find α_R; substituting back in (2.2) then gives h_R. Similarly on the left-hand side:-

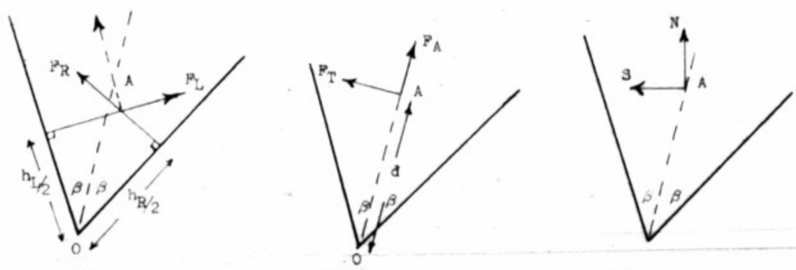
$$h_L \cos(\beta + \epsilon - \theta) - k \cos\theta = h_R \sin(\beta - \theta + \epsilon - \alpha_L) \quad \ldots\ldots \quad (2.4)$$

$$h_L \cos \alpha_L = k \left[\sin(\beta + \epsilon) + \cos(\beta + \epsilon - \alpha_L)\right] \quad \ldots\ldots \quad (2.5)$$

$$\therefore \cos(2\beta - \theta + 2\epsilon - \alpha_L) = -\sin\theta + \frac{\cos\theta . \cos\alpha_L}{1 + \sin\alpha_L} \quad \ldots . \quad (2.6)$$

This, given β, θ and ϵ, determines α_L and then h_L.

The forces on the wedge consist of a uniform normal pressure $2\kappa(1 + \alpha_R)$ along OB, and a normal pressure $2\kappa(1 + \alpha_L)$ along OC. κ is equal to $Y/\sqrt{3}$ where Y is the yield stress of the target material.* There is no frictional force along the sides of the wedge since we are assuming perfect lubrication: it is generally conceded that friction is negligible in dynamic firings, and steps are taken to eliminate it in static punching.



(a) (b) (c)

Fig.3.

The forces exerted on the wedge by the target material are therefore equivalent to two forces F_R, F_L (per unit length of wedge normal to the plane of the paper) acting perpendicularly to the sides at distances $h_R/2$, $h_L/2$ from O. (Fig.3a). We have

$$F_R = 2\kappa\, h_R (1 + \alpha_R) \;;\quad F_L = 2\kappa\, h_L (1 + \alpha_L) \quad \ldots\ldots . \quad (2.7)$$

The resultant of these two forces intersects the axis of symmetry OA at A (say), distance d from O. (Fig.3b). It is convenient to resolve the

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* Work-hardening is neglected. The rate of work-hardening is comparatively small for armour steel, but in any case κ can be regarded as a mean flow stress to be determined by static punching experiments.

resultant force into axial and transverse components F_A, F_T along, and perpendicular to, OA. Then

$$\left.\begin{aligned} F_A &= (F_R + F_L)\sin\beta \\ F_T &= (F_R - F_L)\cos\beta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.8)$$

By taking moments about O:-

$$d = (F_R h_R - F_L h_L)/2F_T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2.9)$$

It is also useful to calculate the component forces N, S in directions normal and tangential to the original plane surface (Fig.3c). These are given by

$$\left.\begin{aligned} N &= F_A\cos(\theta-\epsilon) + F_T\sin(\theta-\epsilon) \\ S &= -F_A\sin(\theta-\epsilon) + F_T\cos(\theta-\epsilon) \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2.10)$$

The forces needed to be applied to the wedge in static punching to keep it moving along OP, and at the given angle of yaw, are simply the reverse of the force system considered above.

It is convenient to express the resistance as a pressure by dividing the forces per unit wedge-length by GH (Fig.2), which represents the diameter of the compression measured in the plane of the original surface. From simple geometry

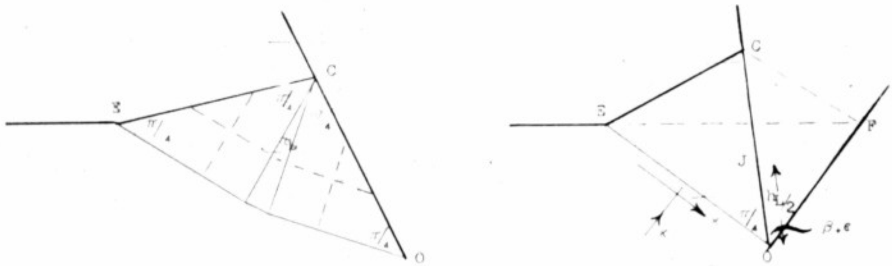
$$D = k\cos\theta\left[\tan(\beta+\theta-\epsilon) + \tan(\beta-\theta+\epsilon)\right] \quad \ldots \quad (2.11)$$

The pressures defined in this way are independent of the depth of penetration and will be represented by small letters (e.g. f_A, f_T, n, s). The frequently used analogue in 3-dimensions is Force/(Area of impression in original surface). As remarked previously the corresponding pressures in 2 or 3 dimensions are found to be roughly equal at normal incidence. Whether this is true at oblique incidence, as seems likely, must be decided by experiment.

It should be noticed that a hole of given shape and orientation below the original surface may be produced in many ways. For example the hole made by an unyawed 30° semi-angle wedge striking at 0° is the same below the surface as that made by the same wedge striking at θ° with θ° positive yaw. The coronets and pressures are however different. In general the same shape of hole below the original surface is produced when $\theta - \epsilon$ is constant (for a given β).

3. Left-hand Coronet not completely plastic

It is now necessary to look more closely into the stress distribution in the left-hand coronet in Fig.2 i.e. the coronet on the side of the wedge furthest away from the original target surface. It was shown in Ref.1. that when $a_L > 0$ and the coronet is completely plastic, the field of slip-lines (or directions of maximum shear stress) takes the form shown in Fig.4.a.



(a) Fig.4 (b)

It consists of two outer regions, stressed uniformly, in which the slip-lines (shown dotted) are orthogonal straight lines meeting the wedge or free surface in $\pi/_4$; and an inner region in which the slip-lines are concentric circular arcs and radii. The maximum shear stress is everywhere constant and equal to κ.

It will be seen that a limit is set to this type of solution by the condition that angle ECO should be greater than or equal to $\pi/_2$ or that $\alpha_L \geq 0$. The critical relation between θ, β and ϵ for which $\alpha_L = 0$ is found from (2.6) to be

$$\cos(2\beta - \theta + 2\epsilon) = -\sin\theta + \cos\theta$$

or

$$\tan\theta = \frac{1 - \cos 2(\beta + \epsilon)}{1 + \sin 2(\beta + \epsilon)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

The solution in Fig.(4a) is valid when θ, β, ϵ are such that

$$\tan\theta < \frac{1 - \cos 2(\beta + \epsilon)}{1 + \sin 2(\beta + \epsilon)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.2)$$

If, for example, we take $\epsilon = 0$, then for $\beta = 30^\circ$ the critical θ is 15°.* For greater obliquities than this the above type of solution is impossible. No such restriction occurs with the coronet on the right-hand or near side of the wedge: the coronet is always completely plastic.

When the relation (3.2) is not satisfied the left-hand coronet is elastic, the plastic strains occurring as material crosses the line OE into the coronet (Fig 4b). The plastic region is in fact localised in the line OE and material becomes stress-released on entering OEC. This rather strange solution is due to our neglect of elastic strains whenever the plastic strains are large in comparison. In a completely accurate solution there would be a plastic region below OE in which plastic and elastic strains would be mostly of the same order, and in this region the large shear strain would take place gradually, becoming increasingly severe as OE is approached. The final state of stress and strain on OE would however differ little from the present solution (the differences being of order $Y/_E$ in comparison; E = Young's Modulus, Y = yield stress).

* When $\beta = 0$, $\pi/_2$ the critical values of θ are 0° and $63^\circ\ 26'$ respectively for zero yaw.

Because of the impossibility of finding the elastic stress distribution in OEC analytically we can only evaluate the resultant force on the wedge face OC and its point of application J.

The equilibrium equation in direction OC is automatically satisfied if we take the normal pressure $= \kappa$ on OE, since there is no tangential force along OC, nor any forces on the free surface EC.

Resolving perpendicularly to OC for the forces acting on region OEC, and taking moments about O, we find the force on the left-hand face of the wedge to be

$$F_L = 2\kappa h_L \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

where

$$OJ = h_L/2 = \frac{\frac{1}{2} \cdot k \cos\theta}{\cos(\beta+\epsilon-\theta) - \sin(\beta+\epsilon-\theta)} \qquad \ldots (3.4)$$

This relation is proved in the Appendix, where it is also shown that angle FCO $= \pi/4$, or

$$OC = k\left\{\sin(\beta+\epsilon) + \cos(\beta+\epsilon)\right\} .$$

The equations (2.8), (2.9), (2.10) hold with the new definition of h_L; the equations involving h_R, α_R are of course unchanged.

4. Numerical Example - Unyawed Wedge striking at various obliquities

For definiteness let us take a wedge of 60° total angle ($\beta = 30^\circ$), and examine the forces on the wedge for several angles of incidence, taking the yaw to be zero. From (3.1) the critical angle of incidence is 15°. For smaller angles than this the equations of Section 3 are valid; for greater angles the equations of Section 4 must be applied to the left-hand coronet.

To begin with we calculate α_R from (2.3), with $\beta = 30^\circ$, $\epsilon = 0^\circ$, for $\theta = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ$. (When $\theta > 60^\circ$ the wedge fails to bite into the surface). Since β occurs only on the left-hand side of (2.3), a convenient way of solving the equation is to take α_R as independent variable and plot the graph of β against α_R for each value of θ.* The value of α_R corresponding to $\beta = 30^\circ$ is then read from the graph. h_R/k is then found from (2.2).

For $\theta = 0^\circ, 10^\circ$ a similar procedure is used to calculate α_L, h_L/k from (2.6) and (2.5). For the higher obliquities h_L/k is found immediately from (3.3). We can then collect the results in Table I. Fig.(6) at the end of the report shows the coronets for $\theta = 0^\circ, 20^\circ$, and 40° for the same depths of penetration normal to the target surface. For comparison for the same k, scaling-down is required for $\theta = 20^\circ, 40^\circ$ in the ratios $\sec 20^\circ = 1.064$, $\sec 40^\circ = 1.305$ respectively.

Table I

θ°	α_R°	α_L°	h_R/k	h_L/k
0	16.5	16.5	1.536	1.536
10	29.0	6.0	1.715	1.421
20	40.6	-	1.953	1.158
30	52.4	-	2.335	0.866
40	64.6	-	3.085	0.661
50	76.9	-	5.220	0.501

* Note that when doing this only values of $\alpha_R > \theta$ give real values of β. When $\beta \to 0$, $\alpha_R \to \theta$.

From (2.7) and Table I F_R can be calculated; F_L is found from (2.7) for $\theta = 0^\circ$, 10°, and from (3.4) for $\theta = 20^\circ, 30^\circ, 40^\circ, 50^\circ$. We divide these by D (2.11) to obtain the quantities f_R, f_L. These values, together with $^d/_k$ from (2.9) are shown in Table II.

Table II

θ	$^D/_k$	$^d/_k$	$^{f_R}/_{2\kappa}$	$^{f_L}/_{2\kappa}$
0	1.155	0.887	1.71	1.71
10	1.185	1.254	2.18	1.32
20	1.286	1.371	2.60	0.90
30	1.500	1.552	2.98	0.58
40	1.970	1.938	3.33	0.34
50	3.412	3.130	3.58	0.15

It will be seen that d is greater than k for all except small obliquities and that $^d/k$ increases rapidly at large obliquities. The reason is clear from a consideration of $^{h_R}/_k$ in Table I and Fig.(6): the right-hand coronet is in contact with the wedge over a length large compared with the depth of penetration.

From (2.8), (2.10) $f_A = {}^{F_A}/D$, $f_T = {}^{F_T}/D$, $n = {}^N/D$, $s = {}^S/D$ are now calculated. (Table III). For ductile armour steel, B.H.N. 230 - 280, an average value of the flow stress Y is 50 tn/in.^{2} and so $2\kappa = 57.7$ tn/in.^{2}. Thus at normal incidence $f_A = 99$ tn/in.^{2}. This entry resistive pressure over the surface impression should be compared with static resistances of over 200 tn/in.^{2}. for deep penetrations when the head and bourrelet are well in the target (Ref.2).

Table III

θ	$^{f_A}/_{2\kappa}$	$^{f_T}/_{2\kappa}$	$^n/_{2\kappa}$	$^s/_{2\kappa}$
0	1.71	0.00	1.71	0.00
10	1.73	0.74	1.85	0.42
20	1.75	1.47	2.15	0.78
30	1.78	2.08	2.58	0.91
40	1.83	2.59	3.07	0.81
50	1.87	2.97	3.48	0.48

It is clear from this Table that an empirical theory, in which the actual pressure distribution is replaced by an equivalent mean hydrostatic pressure acting over the part of the wedge below the original surface, would be unsatisfactory in calculations involving both force components. For the resulting force on the projectile would then consist of a normal pressure over the segment of the original surface intersected by the wedge. The shear component s in Table III would be zero, involving considerable error. Moreover a simple theory of this type would also be inadequate in determining the position of the point A, a knowledge of which is necessary when taking moments. This is due to an underestimate of the length of contact between the wedge and right-hand coronet. It should, perhaps, be pointed out that the difference in the pressure on the two faces of the wedge which is responsible for this large difference may be less for a projectile than for the wedge solution of this report. Freedom for plastic flow round the projectile would tend to equalise this pressure difference.

It is interesting to calculate the work done in making the hole. Since we are not allowing turning of the shot, the transverse force component F_T does no work when $\epsilon = 0$; the whole work being done by the axial component F_A. Since f_A is independent of penetration, the work done per unit length of wedge is $W = Af_A \sec\theta$ where A = area of hole below the original surface $= \frac{1}{2}.D.k \cos\theta$. The work per unit volume of hole is $f_A \sec\theta$. From Table III for a 60° wedge we see that this increases steadily from $3.42\ \kappa$ when $\theta = 0^\circ$ to $5.82\ \kappa$ when $\theta = 50^\circ$; or, taking $Y = 50$ tn/in^2, from 99 tn/in^2 to 168 tn/in^2. If we compare the work expended in penetrating to a given vertical depth $k\cos\theta$ for various obliquities, the increase is still greater since the volume of hole rapidly increases with θ.

This numerical example is sufficient to show the method. Other cases can be treated in a similar way when ad hoc calculations are called for.

Appendix

Stress and Velocity Solution for Coronet when not Completely Plastic

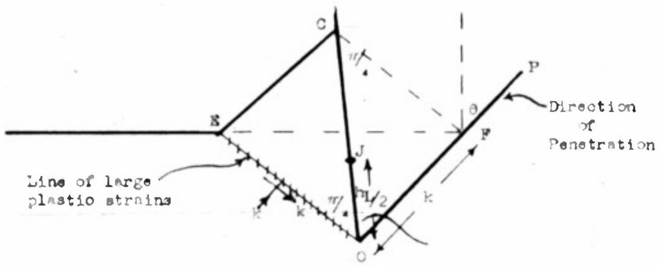


Fig.5.

When angle EOC $< \pi/2$ it is not possible to find a satisfactory solution in which the coronet is completely plastic, and we must therefore look for a solution in which part of the coronet has recovered elastically after being severely strained. Such a solution is one in which the coronet OEC is completely elastic, the plastic region being localised in the slip-line OE which must meet the wedge in $\pi/4$. Along this line the shear acts in the direction shown. Resolving in direction OC shows that the pressure normal to OE (constant since OE is a straight slip-line) is equal to κ. Resolving perpendicularly to OC the resultant force F_L on the wedge is $F_L = \sqrt{2}\,\kappa\,.\,OE$, and by taking moments about O, the point of application is at J where $F_L\,.\,OJ = \frac{1}{2}.\kappa.\,OE^2$. By simple geometry (projecting OE on a line perpendicular to EF)

$$OE \cos\left(\beta + \epsilon + \pi/4 - \theta\right) = k \cos\theta$$

Hence $F_L = 2 k h_L$ where $OJ = h_{L/2}$ and

$$h_L = \frac{k \cos\theta}{\cos(\beta + \epsilon - \theta) - \sin(\beta + \epsilon - \theta)}$$

The stress on OC will be distributed continuously along OC in some way which it is impossible to determine analytically.

Since the coronet moves as a rigid body, with material continually added on the base OE, its velocity is $\sqrt{2}\, V\sin(\beta + \epsilon)$ in the direction OE (V = wedge velocity in direction FO). The material below OE is effectively rigid and so there is a tangential discontinuity in velocity across OE which is permissible since OE is a slip-line and therefore a direction of maximum shear strain-rate.

The shape of EJC has still to be determined from the condition that as the penetration increases EC moves parallel to itself at a rate such that the configuration remains similar. In the wording of Ref. 1 all points in OEC have the same focus, which on the unit diagram with k = 1, is on a line through F parallel to the direction of motion OE and at a distance $\sqrt{2}\sin(\beta + \epsilon)$ from F. It is obvious that this focus must be the tip C of the coronet since elements on the sides EJ, JC remain there and move steadily along these lines in the unit diagram. C is thus determined as the point on the wedge such that angle $FCO = \pi/4$. The final deformation in the coronet can be obtained by simply shearing triangle OEF into triangle OEC.

It is easily verified that angle $OEC > \pi/4$ in this solution so long as (3.2) is not satisfied. When (3.2) is satisfied it would not be valid to use this type of solution in place of the plastic coronet solution of Section 2, since in such cases angle $OEC < \pi/4$. As discussed in detail in Ref.3, because of the maximum shear stress κ acting along EO, the elastic limit would be exceeded in the corner E and the solution would break down. When $OEC > \pi/4$ it is not possible to assert definitely that the elastic limit is not exceeded, without a full elastic stress solution, but from qualitative arguments it seems likely that the coronet will in fact be only elastically stressed.

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2. R. Hill. Theory of Armour Penetration by Tungsten-Carbide Shot and the Effect of Headshape on Performance at High Velocities. Theo. Res. Rep. 29/45

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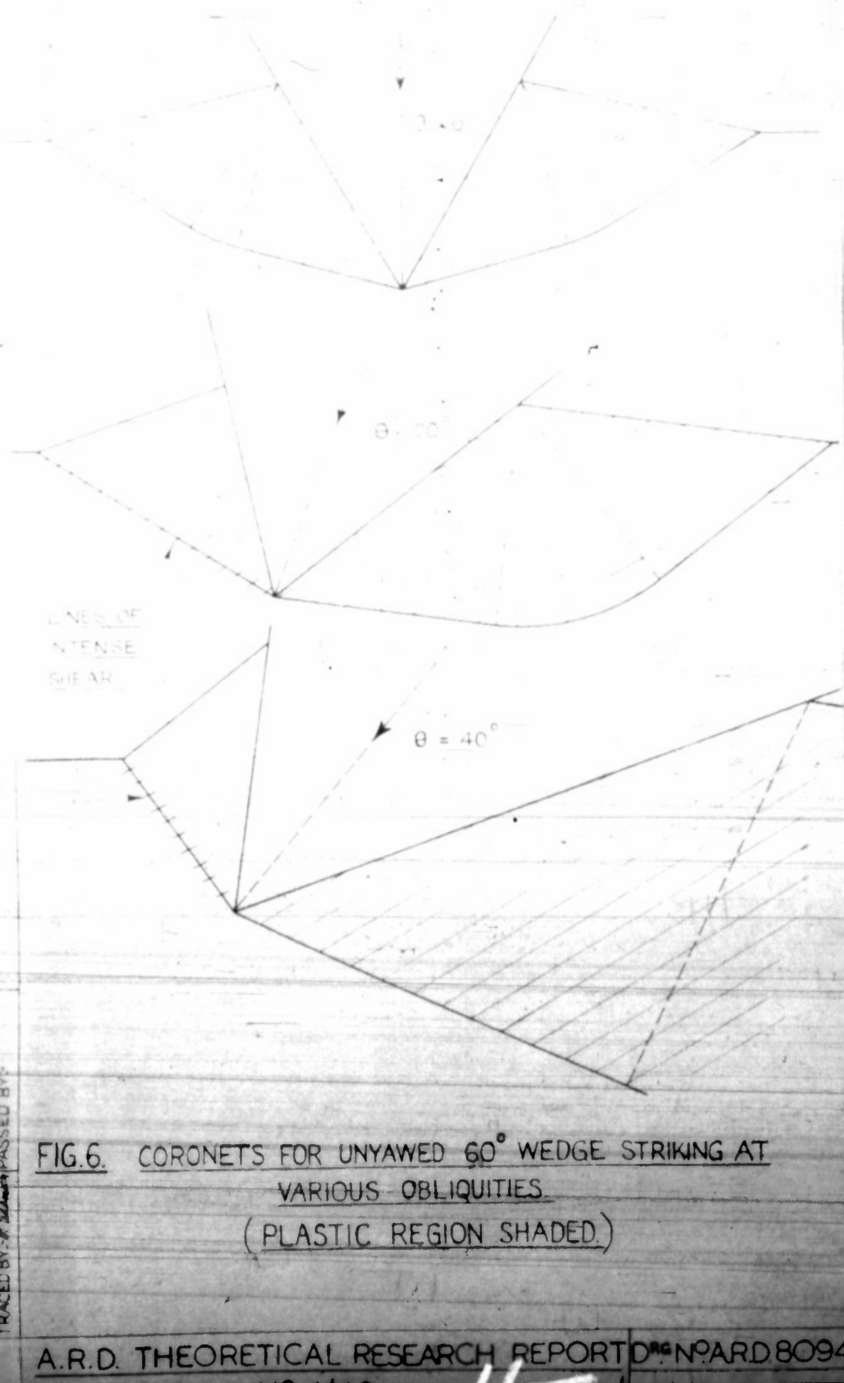

FIG. 6. CORONETS FOR UNYAWED 60° WEDGE STRIKING AT VARIOUS OBLIQUITIES.

(PLASTIC REGION SHADED.)

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TITLE: The Theory of Wedge Penetration at Oblique Incidence and its Application to the Calculation of Forces on a Yawed Shot Impacting on Armour Plate at any Angle AUTHOR(S): Hill, R.; Lee, E. H. ORIGINATING AGENCY: Armament Research Dept., Fort Halstead, Kent PUBLISHED BY: (Same)	ATI- 11261 REVISION (None) ORIG. AGENCY NO. TRR-1/46 PUBLISHING AGENCY NO. (Same)

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